

$$Q. \quad \left\{ \underset{x_1}{(1,0,0)}, \underset{x_2}{(1,1,0)}, \underset{x_3}{(1,1,1)} \right\}$$

$$v_1 = x_1 = (1,0,0)$$

$$v_2 = x_2 - \frac{(v_1 \cdot x_2)}{\|v_1\|^2} v_1 = (1,1,0) - 1 \cdot (1,0,0) = (0,1,0)$$

$$v_3 = x_3 - \frac{(v_1 \cdot x_3)}{\|v_1\|^2} v_1 - \frac{(v_2 \cdot x_3)}{\|v_2\|^2} v_2 = (1,1,1) - (1,0,0) - (0,1,0) = (0,0,1)$$

$$Q: \quad T(1,0,0) = (1,2) \quad T(1,1,0) = (0,2) \quad T: \mathbb{R}^3 \rightarrow \mathbb{R}^2$$

$$T(0,3,0) = ??$$

$$\begin{aligned} (0,3,0) &= \alpha_1 (1,0,0) + \alpha_2 (1,1,0) \\ &= (\alpha_1 + \alpha_2, \alpha_2, 0) \\ \alpha_1 + \alpha_2 &= 0, \quad \alpha_2 = 3 \Rightarrow \alpha_1 = -3 \\ \rightarrow T(0,3,0) &= \alpha_1 T(1,0,0) + \alpha_2 T(1,1,0) \\ &= -3(1,2) + 3(0,2) \\ &= (-3, 0) \end{aligned}$$

$$\text{Span } R(T) = \{ (1,2), (0,2) \}$$

$$= \text{Basis of } R(T)$$

$$\dim R(T) = 2$$

$$\dim V = \dim N(T) + \dim R(T)$$

$$3 = \dim N(T) + 2 \Rightarrow \dim N(T) = 1$$

$$Q.: \quad V = \{ (a,b,c) \mid a-3b-c=0 \}$$

$$\dim V = 2$$

$$V = \{ (a, b, a-3b) \}$$

$$\begin{matrix} a & b \\ 1 & 0 \end{matrix}$$

$$\text{Basis } U = \{ (1, 0, 1), (0, 1, -3) \} \quad \begin{matrix} 0 & 1 \end{matrix}$$

$$U = \{ (a, b, c, d) : a - 3b - d = 0 \}$$

$$\dim U = 3$$

$$U = \{ (a, b, c, a - 3b) \} \quad \begin{matrix} a & b & c \\ 1 & 0 & 0 \end{matrix}$$

$$U = \{ (a, b, c, d, e) : a - 3c - e = 0 \} \quad \begin{matrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix}$$

$$\dim U = 4$$

$$U = \{ (a, b, c, d, a - 3c) \} \quad \begin{matrix} a & b & c & d \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{matrix}$$

$$Q: U = \{ (a, b, c) : a \in \mathbb{R}, b \in \mathbb{I}, c \in \mathbb{Q} \}$$

$$X = (a_1, b_1, c_1) \quad Y = (a_2, b_2, c_2)$$

$$\alpha X + \beta Y = (\alpha a_1 + \beta a_2, \alpha b_1 + \beta b_2, \alpha c_1 + \beta c_2)$$

$$\alpha b_1 + \beta b_2 \in \mathbb{I}$$

$$\alpha c_1 + \beta c_2 \in \mathbb{Q}$$

$$\pi a_1 + e c_2 \notin \mathbb{Q}$$

$$52 \cdot 52 + 55 \cdot 52 = 4 \notin \mathbb{I}.$$

$$Q: U = \{ (x_1, x_2, x_3) : x_1 = 2x_2, \text{ or } x_3 = 3x_2 \}$$

$$x_1 - 2x_2 = 0 \quad \text{or} \quad x_3 - 3x_2 = 0$$

$$\begin{matrix} 0 & \neq 0 \\ (x_1 - 2x_2) & (x_3 - 3x_2) \end{matrix} = 0$$

$$X = (x_1, x_2, x_3) \quad \neq 0$$

$$Y = (y_1, y_2, y_3) \quad (y_1 - 2y_2)(y_3 - 3y_2) = 0$$

$$\alpha X + \beta Y = (\alpha x_1 + \beta y_1, \alpha x_2 + \beta y_2, \alpha x_3 + \beta y_3)$$

$$= (x_1 + y_1, x_2 + y_2, x_3 + y_3)$$

$$[x_1 + y_1 - 2(x_2 + y_2)] \cdot [x_3 + y_3 - 3(x_2 + y_2)] = 0 \quad \text{T.P.}$$

$$[x_1 + y_1 - 2(x_2 + y_2)] \cdot [x_3 + y_3 - 3(x_2 + y_2)] = 0 \quad T.P.$$

$$(x_1 + y_1 - 2x_2 - 2y_2)(x_3 + y_3 - 3x_2 - 3y_2) = 0$$

$$(x_1 - 2x_2 + y_1 - 2y_2)(x_3 - 3x_2 + y_3 - 3y_2) = 0$$

$$\text{LHS: } \underline{(x_1 - 2x_2)(x_3 - 3x_2) + (x_1 - 2x_2)(y_3 - 3y_2) + (y_1 - 2y_2)(x_3 - 3x_2)} \\ + \underline{(y_1 - 2y_2)(y_3 - 3y_2)}$$

Such no. can be chosen if we have $x_1 = 2x_2$ but $x_3 \neq 3x_2$
together with $y_1 \neq 2y_2$ but $y_3 = 3y_2$

$$(x_1, x_2, x_3) = (2, 1, 5)$$

$$(y_1, y_2, y_3) = (0, 1, 3)$$